# Biomechanical model of the shear stress distribution in the femoral neck

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**Abstract**

**Background and Purpose** The shear stress distribution in the femoral neck may be an important biomechanical parameter in the development of slipped capital femoral epiphysis. In our work we present a simple non-invasive method for computation of the shear stress distribution in the femoral neck of an individual hip based on anterior-posterior pelvic radiograph.

**Methods** A mathematical model of the resultant hip force is used and the shear stress tensor is computed according to the procedure for a loaded beam.

**Results** Preliminary results from an illustrative set of three patients with slipped capital femoral epiphysis show that the collum-diaphysis angle, the femoral neck width and the resultant hip force have important influence on shear stress distribution.

**Conclusion** Unfavorable high values of the resultant hip force can be compensated by larger collum-diaphysis angle and wider femoral neck. In further research the method could be used on a larger series of patients to determine the predictive value of the shear stress for different clinical outcomes.

**Keywords:** Femoral Neck; Hip Biomechanics; Shear Stress; Slipped Capital Femoral Epiphysis;

# Introduction

Biomechanical status of hip and femur depends not only on the shape of the bones interacting in the joint but also on the pelvic geometry as a whole [1]. One of the parameters that importantly influence the development of the femur in children and adolescents is the shear stress in the femoral neck and it is considered to be one of the possible elements that contribute to slipped capital femoral epiphysis and *coxa vara* during femoral growth. The aim of our paper is to present a simple, non-invasive method for computation of the shear stress in the femoral neck and to demonstrate the use of the method on an illustrative sample of selected patients, treated for slipped capital femoral epiphysis. The method is based on a previously developed mathematical model for computation of the resultant hip force [1, 2] and on the mechanical method for computation of the stress tensor in a loaded beam [3].

# Materials and Methods

## Computation of the resultant hip force $\boldsymbol{R}$

The resultant hip force was computed with the help of a previously developed mathematical model [1, 2]. The model was derived through the static analysis of one-legged stance that roughly corresponds to the biomechanical status of the hip in slow gait.

The biomechanical model for the computation of the resultant hip force includes the unknown forces of the 9 hip muscles $F_i$ ($i = 1, 2, \ldots, 9$) with known coordinates of the attachment points, known relative areas of the muscle cross-sections $A_i$ and unknown tensions $f_i$, the hip load (the weight of the body $W_B$ minus the weight of the loaded leg $W_L$ originating at the center of the loading mass, and the unknown resultant hip joint force $\boldsymbol{R}$ originating at the femoral head center. The nine model muscles are divided into three muscle groups: anterior, middle and posterior. The average tensions in the particular muscle group are assumed to be equal [1, 2]. The force for each muscle is computed from its unknown average tension $f_i$, its known relative area of the cross-section $A_i$ and its known unit vector in the direction of the muscle force $\mathbf{s_i}$ (derived from the muscle attachment points):

$$\mathbf{F_i} = f_i\, A_i\, \mathbf{s_i}, \quad i = 1, \ldots, 9 \tag{1}$$

In the case of a static one-legged stance, all forces and torques acting in the hip joint are in equilibrium:

$$\sum_{i=1}^{9} \mathbf{F_i} + (\mathbf{W_B} - \mathbf{W_L}) + \mathbf{R} = 0 \tag{2}$$

$$\sum_{i=1}^{9} \mathbf{r_i} \times \mathbf{F_i} + \mathbf{a} \times (\mathbf{W_B} - \mathbf{W_L}) = 0 \tag{3}$$

where $\mathbf{r_i}$ is the known moment arm of the respective muscle and $\mathbf{a}$ is the moment arm of the force $\mathbf{W_B} - \mathbf{W_L}$ which is assumed to lie in the frontal plane of the body. The magnitude of the vector $\mathbf{a}$ is [2]

$$a = \frac{W_B\, c - W_L\, b}{W_B - W_L} \tag{4}$$

where $b = 0.24\, l$ and $c = 0.50\, l$. The solution of the vector equations (2) and (3) yields the three components of the resultant hip force $\boldsymbol{R}$ and the tensions in the abductor muscles.

The three-dimensional reference coordinates of the muscle attachment points were taken from a case study of pelvis in human cadaver [4]. The coordinates of the muscle attachment points

Figure 1: The shear stresses $\tau_{xy}$ and $\tau_{xz}$ in the cross section of the femoral neck. The line of the tangent at $P_1$ intersects the $y$ axis at point D. The resultant shear stress at $P_1$ is directed along $P_1D$ while the resultant shear stress at P is directed along PD.

for an individual patient were estimated by linearly correcting the three-dimensional reference coordinates of the muscle attachment points in the medial-lateral direction and superior-inferior direction with regard to the following radiographic parameters: the interhip distance $l$, pelvic height $H$, pelvic width laterally from the femoral head center $C$ and the coordinates of the insertion point of abductors on the greater trochanter (point $T$) in the frontal plane. The coordinates of the point $T$ ($T_x$ and $T_z$) were measured with respect to the femoral head center.

The radiographic pelvic parameters were measured from the pelvic anterior-posterior radiograph of an individual patient. The contours of the bony structures in a specified radiograph were put into digital form with a digital graphic board. An appropriate computer program HIJOMO [2, 5] was then used to measure the above mentioned radiographic pelvic parameters ($l$, $C$, $H$, $T_x$, $T_z$). Subsequently an appropriate computer program HISTRESS [2, 5] was used to compute the magnitude and the direction of the resultant hip joint force $R$ in a static one-legged stance in accordance with the above described procedure.

## Computation of the stress tensor $\sigma$

We used a simple mechanical model to compute the stress tensor in the femoral neck. In our computation, the force of acetabulum on the hip neck was substituted by the hip joint force $\boldsymbol{R}$ with the application point in the center of the femoral head. The femoral neck and the femoral head were assumed to form a beam of isotropic material that is fixed on the lateral side and free on the medial side. The rest of the femur can be considered as the wall into which the femoral neck is fixed. The cross-section of the femoral neck and the femoral head was assumed to have the circular form with the radius $a$. The origin of the coordinate system was chosen in the center of the cross-section of femoral neck (Fig. 1). In the case of a static one-legged stance, the force $\boldsymbol{R}$ lies in the frontal plane of the body and therefore has only two components in the given coordinate system, the longitudinal component -$P$ and the transverse component $F_s$. The component -$P$ compresses the bone in the negative direction of the $x$ axis while the component $F_s$ bends the bone in the direction of the $y$ axis.

The stress tensor

$$\sigma = \begin{pmatrix} \sigma_{xy} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & 0 & 0 \\ \tau_{zx} & 0 & 0 \end{pmatrix} \tag{5}$$

gives information about the stress in the femoral neck. The stress tensor is symmetric and has in general only six independent components. The components $\sigma_y$, $\sigma_z$, $\tau_{yz}$ and $\tau_{zy}$ are assumed to be zero. The stress tension $\sigma_x$ along the $x$ axis is calculated using the stress calculation method of the beam loaded on one side. After some calculations we get

$$\sigma_x = -\frac{P}{\pi\, a^2} + F_s\,(x - x_R)\,\frac{4\,y}{\pi\, a^4} \quad , \quad x = x_R \tag{6}$$

The first term in the equation represents the stress in the direction of the $x$ axis as the consequence of the compressive stress of the longitudinal component of the resultant hip force -$P$. The second term in the equation represents the stress in the direction of the $x$ axis caused by the bending (compressive and tensile stresses) of the femoral neck as a result of the transversal hip joint force $F_s$. The $x$ coordinate of the application point of the force $\boldsymbol{R}$ is denoted by $x_R$.

Apart from pure compression and pure bending we also considered the components of the shear stress tensor in the femoral neck, $\tau_{xy}$ and $\tau_{xz}$. At small deflections, the shear stress components are small in comparison with the stress component $\sigma_x$. The elementary beam theory yields an approximate shear stress solution [3]. We assume the shear force to be directed along the $y$ axis and to be equal to $F_s$. Therefore, the resultant $\tau_{xy}$ and $\tau_{xz}$ integrated over the whole cross section would have to equal $F_s$. Since the resultant shear stress at the point on the boundary condition of the section is tangential to the boundary condition, and its ratio to the resultant shear stress at P is $\mathrm{P_1D/PD}$ (Fig. 1) it follows:

$$\frac{\tau_{z1x}}{\tau_{xy}} = \frac{\mathrm{d}z_1}{\mathrm{d}y} \quad , \qquad \frac{\tau_{z1x}}{\tau_{zx}} = \frac{z_1}{z} \tag{7}$$

where $\tau_{z1x}$ is the shear stress component at $z_1$, $\tau_{zx}$ is the shear stress component at $z$. The point P has coordinate $(y, z)$ while point $\mathrm{P_1}$ has coordinate $(y, z_1)$.

The equations (7) can be combined into equation

$$\tau_{zx} = \frac{z\,\mathrm{d}z_1}{z_1\,\mathrm{d}y}\,\tau_{xy} \tag{8}$$

The expression (8) for the $\tau_{zx}$ is substituted in the equilibrium relation

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \tag{9}$$

which represents the necessary conditions for the components of the resultant force acting on the femoral head to vanish. The body force in the direction of $x$ axis is assumed to be zero. Inserting Eq. (8) into Eq. (9) we get

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{z_1}\frac{\mathrm{d}z_1}{\mathrm{d}y}\,\tau_{xy} = 0 \tag{10}$$

Eq. (10) can be transformed into

$$\frac{\partial \sigma_x}{\partial x} + \frac{1}{z_1}\frac{\partial}{\partial y}\,(z_1\,\tau_{xy}) = 0 \tag{11}$$

By inserting Eq. (6) into Eq. (11) and solving differential equation, we get the resultant shear stress components

$$\tau_{xy} = 4\,F_s\,\frac{a^2 - y^2}{3\,\pi\,a^4} \tag{12}$$

$$\tau_{xz} = -4\,F_s\,\frac{y\,z}{3\,\pi\,a^4} \tag{13}$$

The resultant shear stress at any point in the cross section is

$$\tau = \sqrt{\tau_{xy}^2 + \tau_{xz}^2} = 4\,F_s\,\frac{(a^2 - y^2)^2 + y^2\,z^2}{3\,\pi\,a^4} \tag{14}$$

The maximum of the function in Eq.14, i.e. the maximal shear stress acting in the femoral neck, is then computed as:

$$\tau_{max} = \frac{4\,F_s}{3\,\pi\,a^2} \tag{15}$$

The force $F_s$ is a function of the coordinate $x$ and the resultant shear stress varies from section to section. The force $F_s$ was computed from the magnitude and inclination of the resultant force $\boldsymbol{R}$ and from the femoral collum-diaphysis (CCD) angle. The inclination of the femoral diaphysis with respect to the vertical was assumed to be 6° in all patients.

| Patient No. | CCD [°] | $R/W_B$ [1] | *Inclination of* **R** [°] | $F_s/W_B$ [1] | $-P/W_B$ [1] | $a$ [mm] | $x - x_R$ [mm] | $\tau_{max}/W_B$ [dm$^{-2}$] |
|---|---|---|---|---|---|---|---|---|
| 1 | 112 | 2.54 | 9.2 | 2.03 | 1.54 | 19 | 18 | 24.0 |
| 2 | 138 | 2.42 | 11.3 | 1.01 | 2.20 | 18 | 20 | 13.3 |
| 3 | 150 | 2.83 | 6.3 | 0.86 | 2.70 | 17 | 17 | 13.4 |

Table 1: Maximal shear stress $\tau_{max}/W_B$ for each patient with the corresponding parameters.

# Results and Discussion

The presented method was tested on an illustrative sample of three selected patients (Table 1), who were operated upon at the Department of Orthopaedic Surgery, University Medical Center Ljubljana due to slipped capital femoral epiphysis. For each patient the values of $l$, $H$, $C$, $T_x$, $T_z$, femoral collum-diaphysis (CCD) angle, the most narrow transverse diameter of the femoral neck $a$ and the distance of the plane of $a$ to the center of the femoral head $x$-$x_R$ were measured from the anterior-posterior radiogram. From this data the resultant hip force and the vertical inclination of the resultant force were first computed and used in the equation (15) to finally compute the maximal shear stress in the narrowest part of the femoral neck. All the parameters were normalized with regard to the subjects' weight $W_B$ to allow for comparison between the subjects. The three exemplary cases in Table 1 show a very wide range of the shear stress values. The patients did not differ considerably in the width of the femoral neck $a$. Patient 1 had the highest value of shear stress and the lowest value of the CCD angle. Patients 2 and 3 had similar values of the shear stress despite different values of $R/W_B$ - this can only be explained through different values of the CCD angle, where patient 3 compensated its unfavorably high value of the resultant hip force with larger CCD angle.

The method could be used as a useful non-invasive tool to investigate shear stresses in the femoral neck. The advantages of the method include its simplicity, non-invasive measuring technique and the ability to perform retrograde studies on the basis of archive radiograms. One of the disadvantages of the method is the relatively simplified procedure for the determination of the resultant hip force which corresponds only to one-legged stance and slow gait. This disadvantage could be improved by determining the resultant hip force through other means [6] (e.g. laboratory measurements on a piezoelectric plate) and in more complex situations [7]. In this paper we have only shown examples of the shear stress analysis in the transverse narrowest part of the femoral neck. Appropriate measurements in other planes and parts of the neck would make the evaluation of stresses in other relevant planes possible.

# Conclusion

The resultant hip force, the CCD angle and the femoral neck width are important factors that influence the shear stress distribution. Unfavorably high values of the resultant hip force can be compensated by larger collum-diaphysis angle and wider femoral neck. It remains to be investigated further, which of the included parameters most importantly influence the values of the maximal shear stress in the femoral neck in different populations of healthy or pathological hips. Such data could further help in investigating and comparing and/or simulating the biomechanical outcomes of different surgical approaches with the purpose of finding the optimal operational procedure for a given patient.

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